

Matthie Recalculation Path

Number Sense: A Complete Course (Book 1) Teacher Guide & Keys

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Built against Master Framework v1.1

Teacher Guide & Keys

This guide covers every skill in the Number Sense skill map (24 skills across Chapters 1-10). Each skill entry gives the mathematical claim, a Route Finder item, a Tripwire item that discriminates the skill's dominant confusion, a Confirmation item in a different surface form, a Recalculation Check, and a Road Test that checks transfer without naming the chapter -- plus the full teacher pathway: observed response, likely reasoning (a hypothesis, not a diagnosis), how to confirm it, the teaching move, and where to route next.

A single wrong answer is a hypothesis, not a diagnosis. Confirm with a second independent piece of evidence before treating a pattern as established.

Teacher Desk Card

Keep this page open while working through any skill below. Four rules, one per stage of the cycle:

FIND: STOP FINDING when the evidence is sufficient to choose a teaching target. Do not measure everything the student cannot do.

TEACH: STOP RETEACHING the wider unit when the confirmed gap is narrower. Address the smallest idea actually blocking progress.

CHECK: STOP TESTING when the student has provided sufficient independent evidence. A fresh problem or existing classwork both count.

MOVE: STOP WORKING THE OLD TARGET once the student has met the defined mastery criterion. Record status and route to what's next.

Status codes: NY (Not Yet) -- R (Recalculating) -- M (Mastered) -- IE (Insufficient Evidence). Fixed across the whole series.

The lightest-evidence rule: do not administer a separate diagnostic form when reliable, current classwork already answers the question.

Quick Answer Key (Student Forms)

For fast grading against a completed Student Form. Organized in the same order as Student Forms -- Route Finder, Tripwire, Confirmation, Road Test, per skill. For the reasoning, the reteach move, and where to route a student next, see that skill's full entry later in this guide.

Form ID	Item	Correct response
NS-PV-01-RF1	Route Finder	0.7 ($0.7 = 0.70$, and 70 hundredths is more than 65 hundredths).
NS-PV-01-RF1	Tripwire	0.6 is greater ($0.6 = 0.60$, and 60 hundredths is more than 58 hundredths).
NS-PV-01-CF1	Confirmation	They are equal -- a trailing zero does not change a decimal's value.
NS-PV-01-CF1	Road Test	12.38 seconds is faster (smaller time); $12.4 = 12.40$, and 40 hundredths is more than 38 hundredths.
NS-PV-02-RF1	Route Finder	three thousand, four hundred six.
NS-PV-02-RF1	Tripwire	four thousand, two hundred eight -- the 0 in the hundreds place must still be read as an empty hundreds column, not skipped.
NS-PV-02-CF1	Confirmation	ninety thousand, one hundred five.
NS-PV-02-CF1	Road Test	8,006.
NS-PV-03-RF1	Route Finder	4,800 (the tens digit, 3, is less than 5).
NS-PV-03-RF1	Tripwire	27,000 (the hundreds digit, 7, forces a round up).
NS-PV-03-CF1	Confirmation	18,000 (the hundreds digit, 4, is less than 5).
NS-PV-03-CF1	Road Test	\$7,000.
NS-FM-01-RF1	Route Finder	1, 2, 3, 4, 6, 12.
NS-FM-01-RF1	Tripwire	1, 2, 3, 6 -- a short list of numbers that divide into 6, not numbers counted out from it.
NS-FM-01-CF1	Confirmation	Multiples: 9, 18, 27, 36, 45. Factors: 1, 3, 9.
NS-FM-01-CF1	Road Test	1, 3, 5, 15 rows would work evenly (the factors of 15).
NS-FM-02-RF1	Route Finder	No. $51 = 3 \times 17$, so it's composite.
NS-FM-02-RF1	Tripwire	No. $87 = 3 \times 29$, so it's composite.

Form ID	Item	Correct response
NS-FM-02-CF1	Confirmation	Yes -- testing 2,3,5,7 (up to $\sqrt{97} \approx 9.8$), none divide evenly.
NS-FM-02-CF1	Road Test	No -- $57 = 3 \times 19$, so it's composite, not prime.
NS-FM-03-RF1	Route Finder	6 m, the GCF of 12 and 18.
NS-FM-03-RF1	Tripwire	24 minutes, the LCM of 8 and 12 -- this is a repeating-event situation, which needs LCM, not GCF.
NS-FM-03-CF1	Confirmation	60 seconds (LCM of 15 and 20).
NS-FM-03-CF1	Road Test	12 items per box (GCF of 24 and 36).
NS-FR-01-RF1	Route Finder	$\frac{2}{3}$ (common denominator 15: $\frac{10}{15}$ vs. $\frac{9}{15}$).
NS-FR-01-RF1	Tripwire	$\frac{3}{5}$ is greater (common denominator 35: $\frac{21}{35}$ vs. $\frac{20}{35}$).
NS-FR-01-CF1	Confirmation	$\frac{5}{11}$ (common denominator 99: $\frac{44}{99}$ vs. $\frac{45}{99}$).
NS-FR-01-CF1	Road Test	$\frac{2}{5}$ cup uses more (common denominator 40: $\frac{15}{40}$ vs. $\frac{16}{40}$).
NS-FR-02-RF1	Route Finder	$\frac{8}{20}$ (multiply both parts by 4).
NS-FR-02-RF1	Tripwire	$\frac{9}{12}$ (multiply both the numerator and denominator by 3).
NS-FR-02-CF1	Confirmation	$\frac{20}{24}$ (multiply both parts by 4).
NS-FR-02-CF1	Road Test	$\frac{12}{32}$ (multiply both parts by 4).
NS-FR-03-RF1	Route Finder	$\frac{3}{4}$ (GCF = 6).

Form ID	Item	Correct response
NS-FR-03-RF1	Tripwire	$\frac{2}{3}$ (GCF of 12 and 18 is 6, not 2; $\frac{6}{9}$ still shares a common factor of 3).
NS-FR-03-CF1	Confirmation	$\frac{2}{3}$ (GCF=10). Yes, $\frac{9}{4}$ is a valid, correctly-formed answer as an improper fraction.
NS-FR-03-CF1	Road Test	$\frac{2}{3}$.
NS-DC-01-RF1	Route Finder	3.12 (2 total decimal places).
NS-DC-01-RF1	Tripwire	5.6 (2 total decimal places from both factors: one from 3.5, one from 1.6).
NS-DC-01-CF1	Confirmation	0.30 (2 total decimal places).
NS-DC-01-CF1	Road Test	3.5 cups.
NS-DC-02-RF1	Route Finder	4.75 (write 8 as 8.00 first).
NS-DC-02-RF1	Tripwire	4.54 (write 5 as 5.00, then subtract).
NS-DC-02-CF1	Confirmation	8.25.
NS-DC-02-CF1	Road Test	\$6.60.
NS-DC-03-RF1	Route Finder	12 (both numbers $\times 10$: $84 \div 7 = 12$). Estimate: about $8 \div 1 = 8$, so 12 is reasonable for dividing by less than 1.
NS-DC-03-RF1	Tripwire	7 (both numbers shifted by $\times 10$: $63 \div 9 = 7$).
NS-DC-03-CF1	Confirmation	9; bigger than 5.4, since dividing by less than 1 increases the result.
NS-DC-03-CF1	Road Test	8 pieces.
NS-IN-01-RF1	Route Finder	-3 (it sits closer to zero, farther right on the number line).
NS-IN-01-RF1	Tripwire	-6 is greater. It sits closer to zero, so it is farther right on the number line.
NS-IN-01-CF1	Confirmation	-11, -4, -1.
NS-IN-01-CF1	Road Test	-15 degrees is colder (farther from zero on the negative side, i.e., the lesser value).
NS-IN-02-RF1	Route Finder	$ -9 = 9$; the opposite of -9 is 9.
NS-IN-02-RF1	Tripwire	$ -7 = 7$ -- absolute value measures distance, and distance cannot be negative.

Form ID	Item	Correct response
NS-IN-02-CF1	Confirmation	Opposite of 12 is -12. $ -12 = 12$.
NS-IN-02-CF1	Road Test	85 meters below the surface (the absolute value of -85).
NS-IN-03-RF1	Route Finder	-8 (same signs: add the sizes, keep the negative sign).
NS-IN-03-RF1	Tripwire	12 (rewrite as $7 + 5$).
NS-IN-03-CF1	Confirmation	-11; and 13.
NS-IN-03-CF1	Road Test	-35 m.
NS-IN-04-RF1	Route Finder	24 (two negatives multiplied give a positive).
NS-IN-04-RF1	Tripwire	-11 (same signs added get bigger and stay negative, not positive).
NS-IN-04-CF1	Confirmation	18; and -18.
NS-IN-04-CF1	Road Test	-20 points.
NS-RN-01-RF1	Route Finder	Yes -- $-4 = -\frac{4}{1}$. Yes -- 0.121212... equals a fraction of two integers ($\frac{4}{33}$).
NS-RN-01-RF1	Tripwire	8 is rational -- every integer can be written over 1 ($\frac{8}{1}$), which makes it rational.
NS-RN-01-CF1	Confirmation	Yes, $-6 = -\frac{6}{1}$. No, a non-repeating, non-terminating decimal is irrational.
NS-RN-01-CF1	Road Test	Yes -- $5 = \frac{5}{1}$, so it's rational.
NS-RN-02-RF1	Route Finder	$\frac{3}{4}$ is greater ($\frac{3}{4} = 0.75$, which is greater than 0.7).
NS-RN-02-RF1	Tripwire	-0.35 is greater ($-\frac{2}{5} = -0.4$, and -0.35 is closer to zero than -0.4).
NS-RN-02-CF1	Confirmation	$\frac{5}{6}$ is greater ($\frac{5}{6} \approx 0.833$, which is greater than 0.8).
NS-RN-02-CF1	Road Test	0.4 off is the bigger discount ($\frac{3}{8} = 0.375$, which is less than 0.4).

Form ID	Item	Correct response
NS-OP-01-RF1	Route Finder	12 (division and multiplication share equal priority, work left to right: $24 \div 4 = 6$, $6 \times 2 = 12$).
NS-OP-01-RF1	Tripwire	24 (work left to right: $30 \div 5 = 6$, $6 \times 4 = 24$).
NS-OP-01-CF1	Confirmation	14 (addition and subtraction, left to right: $16 - 5 = 11$, $11 + 3 = 14$).
NS-OP-01-CF1	Road Test	15 ($40 \div 8 = 5$, $5 \times 3 = 15$).
NS-OP-02-RF1	Route Finder	15 (bracket first: $9 - 4 = 5$, then $5 \times 3 = 15$).
NS-OP-02-RF1	Tripwire	16 (evaluate the exponent first: $5^2 = 25$, then $25 - 9 = 16$).
NS-OP-02-CF1	Confirmation	12 (bracket first: $2 + 1 = 3$, then exponent: $6^2 = 36$, then $36 \div 3 = 12$).
NS-OP-02-CF1	Road Test	13 square meters ($4^2 = 16$, then $16 - 3 = 13$).
NS-PW-01-RF1	Route Finder	64 ($4 \times 4 \times 4$).
NS-PW-01-RF1	Tripwire	125 ($5 \times 5 \times 5$).
NS-PW-01-CF1	Confirmation	81 ($3 \times 3 \times 3 \times 3$).
NS-PW-01-CF1	Road Test	64.
NS-PW-02-RF1	Route Finder	$7^0 = 1$. $-3^2 = -9$ (base is 3). $(-3)^2 = 9$ (base is -3).
NS-PW-02-RF1	Tripwire	$-4^2 = -16$ (base is 4, then negated) and $(-4)^2 = 16$ (base is -4).
NS-PW-02-CF1	Confirmation	$5^0 = 1$. $-2^4 = -16$ (base is 2). $(-2)^4 = 16$ (base is -2).
NS-PW-02-CF1	Road Test	No -- same base (3), exponents add: $3^2 \times 3^4 = 3^6 = 729$, not 9^6 .
NS-PW-03-RF1	Route Finder	7 ($7 \times 7 = 49$).
NS-PW-03-RF1	Tripwire	$\sqrt{50}$ is between 7 and 8 (not a perfect square); $\sqrt{50} \neq 25$, since $25 \times 25 = 625$, not 50.
NS-PW-03-CF1	Confirmation	$\sqrt{121} = 11$. $\sqrt{40} \neq 20$ since $20 \times 20 = 400$, not 40; square root needs equal factors, not halving.
NS-PW-03-CF1	Road Test	6 inches ($\sqrt{36}$).
NS-PW-04-RF1	Route Finder	$8 < \sqrt{70} < 9$ ($8^2 = 64$, $9^2 = 81$), approximately 8.4.
NS-PW-04-RF1	Tripwire	$12 < \sqrt{155} < 13$ ($12^2 = 144$, $13^2 = 169$), approximately 12.4.
NS-PW-04-CF1	Confirmation	$14 < \sqrt{200} < 15$ ($14^2 = 196$, $15^2 = 225$). $\sqrt{49} = 7$ only, not ± 7 .

Form ID	Item	Correct response
NS-PW-04-CF1	Road Test	Between 11 and 12 meters ($11^2=121$, $12^2=144$), approximately 11.4.

Chapter 1 -- Place Value, Number Forms, and Comparing Numbers

NS-PV-01 Compare decimals using place-value alignment, not digit count

Mathematical claim: Comparing decimals requires lining up the decimal point and comparing one place at a time from the left; more digits after the decimal point does not mean a larger value, and a trailing zero never changes a decimal's value.

Route Finder

Item: Which is greater: 0.7 or 0.65?

Correct response: 0.7 ($0.7 = 0.70$, and 70 hundredths is more than 65 hundredths).

Tripwire

Observed: A student compares 0.6 and 0.58 and says 0.58 is greater because it has more digits.

Likely wrong response: 0.58, from assuming more decimal digits means a bigger number.

Correct response: 0.6 is greater ($0.6 = 0.60$, and 60 hundredths is more than 58 hundredths).

Likely reasoning: may indicate longer-is-larger thinking: an answer that picks the decimal with more digits after the point, regardless of actual size, is the book's own named diagnostic trigger for this error

Confirm (approach): Give a second decimal comparison with a different digit-count pattern. If the longer decimal is chosen again regardless of value, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Which is greater: 0.35 or 0.350?

Correct response: They are equal -- a trailing zero does not change a decimal's value.

Teach: Line up both numbers at the decimal point on a place-value chart before comparing any digit, starting from the left.

Fresh Check (Recalculation Check)

Item: Which is greater: 0.42 or 0.4?

Correct response: They are equal in the tenths place; 0.42 is greater ($0.4 = 0.40$, and 42 hundredths is more than 40 hundredths).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two race times are recorded as 12.4 seconds and 12.38 seconds, and a coach asks which runner was faster, with no place-value instructions given.

Correct response: 12.38 seconds is faster (smaller time); $12.4 = 12.40$, and 40 hundredths is more than 38 hundredths.

Route: If check passes: move to NS-PV-02. If check fails: reteach with the place-value chart until alignment is automatic.

NS-PV-02 Read and write numbers correctly across word, expanded, and standard form

Mathematical claim: Every place in a number is read and written, including a placeholder zero in the middle; word form uses words only, expanded form uses numbers and addition only, and the two are never mixed.

Prerequisites: NS-PV-01

Route Finder

Item: Write 3,406 in word form.

Correct response: three thousand, four hundred six.

Tripwire

Observed: A student writes 4,208 in word form as 'four thousand, twenty-eight.'

Likely wrong response: four thousand, twenty-eight, from skipping the placeholder zero as if it held no value.

Correct response: four thousand, two hundred eight -- the 0 in the hundreds place must still be read as an empty hundreds column, not skipped.

Likely reasoning: may indicate a placeholder zero inside a whole number is being skipped rather than read as a value-holding column, exactly the book's own diagnostic trigger for this error

Confirm (approach): Give a second number with a placeholder zero in a different position. If it's skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Write 90,105 in word form.

Correct response: ninety thousand, one hundred five.

Teach: Read column by column, out loud, every place including zeros: thousands, hundreds, tens, ones.

Fresh Check (Recalculation Check)

Item: Write 7,050 in word form, and write $6,000 + 200 + 9$ in standard form.

Correct response: seven thousand, fifty; and 6,209.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A store's inventory count is written as 'eight thousand, six' and a clerk needs to enter it as a numeral, with no place-value terms used in the request.

Correct response: 8,006.

Route: If check passes: move to NS-PV-03. If check fails: reteach column-by-column reading with zeros included.

NS-PV-03 Round from the digit to the right of the target place

Mathematical claim: Rounding decisions are made by checking the single digit immediately to the right of the target place, not the target digit itself.

Prerequisites: NS-PV-01

Route Finder

Item: Round 4,832 to the nearest hundred.

Correct response: 4,800 (the tens digit, 3, is less than 5).

Tripwire

Observed: A student rounds 26,749 to the nearest thousand and answers 26,000.

Likely wrong response: 26,000, from checking the thousands digit itself instead of the digit to its right.

Correct response: 27,000 (the hundreds digit, 7, forces a round up).

Likely reasoning: may indicate the target place's own digit is being checked instead of the digit to its right, the book's own named diagnostic trigger for this error

Confirm (approach): Give a second rounding item at a different place value. If the wrong digit is checked again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Round 18,459 to the nearest thousand.

Correct response: 18,000 (the hundreds digit, 4, is less than 5).

Teach: Circle the digit directly to the right of the target place before deciding up or down; that single digit makes the whole decision.

Fresh Check (Recalculation Check)

Item: Round 5,362 to the nearest hundred.

Correct response: 5,400 (the tens digit, 6, forces a round up).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A school fundraiser total of \$6,750 needs to be reported to the nearest thousand dollars, with no rounding instructions given.

Correct response: \$7,000.

Route: If check passes: chapter secure, move to NS-FM-01 (Chapter 2). If check fails: reteach circling the digit to the right before every rounding decision.

Chapter 2 -- Factors, Multiples, Primes, and Divisibility

NS-FM-01 Distinguish factors from multiples

Mathematical claim: Factors divide into a number and form a short, finite list; multiples count out from a number and form an endless list. The two describe opposite directions of the same multiplying relationship.

Route Finder

Item: List the factors of 12.

Correct response: 1, 2, 3, 4, 6, 12.

Tripwire

Observed: A student is asked for the factors of 6 and lists 6, 12, 18, 24.

Likely wrong response: 6, 12, 18, 24, a list of multiples given where factors were asked for.

Correct response: 1, 2, 3, 6 -- a short list of numbers that divide into 6, not numbers counted out from it.

Likely reasoning: *may indicate the direction of the multiplying relationship is being lost: a list that keeps growing past the number itself, offered as factors, is the book's own diagnostic trigger for this error*

Confirm (approach): Give a second item asking for multiples where factors were the more natural answer, or vice versa. If direction is confused again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: List the multiples of 9 up to 45, then list the factors of 9.

Correct response: Multiples: 9,18,27,36,45. Factors: 1,3,9.

Teach: Factors divide INTO a number and stop. Multiples come OUT from a number by counting up and never stop. Name which direction the question asks before answering.

Fresh Check (Recalculation Check)

Item: List the factors of 20, then list the first four multiples of 20.

Correct response: Factors: 1, 2, 4, 5, 10, 20. Multiples: 20, 40, 60, 80.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A teacher wants to arrange 15 desks and asks what numbers of rows would divide the desks evenly, with no factor/multiple language used.

Correct response: 1,3,5,15 rows would work evenly (the factors of 15).

Route: If check passes: move to NS-FM-02. If check fails: reteach the divides-into vs. counts-out-from distinction explicitly.

NS-FM-02 Test primality correctly, including 1 and odd numbers

Mathematical claim: A prime number has exactly two factors; testing primality requires checking every prime up to the square root of the number, not stopping after ruling out 2, 5, and 10. The number 1 has exactly one factor, so it is neither prime nor composite.

Prerequisites: NS-FM-01

Route Finder

Item: Is 51 prime?

Correct response: No. $51 = 3 \times 17$, so it's composite.

Tripwire

Observed: A student is asked if 87 is prime and says yes, because it's odd and doesn't end in 0, 2, 4, 5, 6, or 8.

Likely wrong response: Yes, from stopping the divisibility test after only the most obvious rules without testing 3.

Correct response: No. $87 = 3 \times 29$, so it's composite.

Likely reasoning: may indicate a prime classification is being made without testing every prime up to the square root, the book's own diagnostic trigger for this error (odd-numbers-are-prime assumption)

Confirm (approach): Give a second borderline composite number that is odd and doesn't obviously fail small divisibility tests. If misclassified again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Is 97 prime? Show your divisibility tests.

Correct response: Yes -- testing 2,3,5,7 (up to $\sqrt{97} \approx 9.8$), none divide evenly.

Teach: Test every prime up to the square root of the number (2, 3, 5, 7 for anything under 100). Stop only once none of them divide evenly.

Fresh Check (Recalculation Check)

Item: Is 1 prime? Is 91 prime?

Correct response: 1 is neither prime nor composite (exactly one factor). $91 = 7 \times 13$, so it's composite, not prime.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A locker combination uses a number that a student claims can't be split into any smaller whole-number groups except itself and 1, and asks if 57 qualifies, with no prime-testing instructions given.

Correct response: No -- $57 = 3 \times 19$, so it's composite, not prime.

Route: If check passes: move to NS-FM-03. If check fails: reteach the square-root divisibility test and the 1-is-neither rule.

NS-FM-03 Find complete factor pairs and choose GCF or LCM by context

Mathematical claim: Every factor arrives with a matching partner; a factor list is incomplete without it. Choosing between GCF and LCM depends on what the situation is doing, not a memorized keyword: splitting into equal groups needs GCF, waiting for two repeating events to align needs LCM.

Prerequisites: NS-FM-02

Route Finder

Item: You need to cut 12 m and 18 m of rope into the longest equal pieces possible, with none left over. How long should each piece be?

Correct response: 6 m, the GCF of 12 and 18.

Tripwire

Observed: Two buses leave the station at the same time, one every 8 minutes and one every 12 minutes. A student is asked when they'll next leave together and computes the GCF, getting 4 minutes.

Likely wrong response: 4 minutes, from picking GCF for a situation that needs two cycles to align.

Correct response: 24 minutes, the LCM of 8 and 12 -- this is a repeating-event situation, which needs LCM, not GCF.

Likely reasoning: *may indicate GCF or LCM is being chosen by keyword pattern-matching rather than reasoning about whether the situation splits into equal groups or waits for cycles to align, the book's own diagnostic trigger for this error*

Confirm (approach): Give a second word problem of the opposite type (equal-grouping when the first was repeating-events, or vice versa). If the wrong operation is chosen again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Two lighthouses flash every 15 seconds and every 20 seconds. If they flash together now, when do they next flash together?

Correct response: 60 seconds (LCM of 15 and 20).

Teach: Ask what the problem is actually doing: splitting into the largest equal pieces means GCF; waiting for two repeating events to land together again means LCM.

Fresh Check (Recalculation Check)

Item: List all factor pairs of 36, then find the LCM of 6 and 9.

Correct response: Factor pairs of 36: (1,36),(2,18),(3,12),(4,9),(6,6). LCM of 6 and 9: 18.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A baker wants to divide 24 cupcakes and 36 cookies into identical gift boxes with the most items per box and nothing left over, with no GCF/LCM language used.

Correct response: 12 items per box (GCF of 24 and 36).

Route: If check passes: chapter secure, move to NS-FR-01 (Chapter 3). If check fails: reteach asking what the situation is doing before choosing GCF or LCM.

Chapter 3 -- Fraction Foundations

NS-FR-01 Compare fractions using a common denominator or benchmark, not denominator size

Mathematical claim: A larger denominator means smaller pieces, not a larger fraction; comparing fractions requires a method (common denominator, matching numerators, or a benchmark like one-half), not a guess based on which denominator looks bigger.

Route Finder

Item: Which is greater: $\frac{2}{3}$ or $\frac{3}{5}$?

Correct response: $\frac{2}{3}$ (common denominator 15: $\frac{10}{15}$ vs. $\frac{9}{15}$).

Tripwire

Observed: A student compares $\frac{3}{5}$ and $\frac{4}{7}$ and says $\frac{4}{7}$ is greater because 7 is the bigger denominator.

Likely wrong response: $\frac{4}{7}$, from assuming the bigger denominator means the bigger fraction, with no common-denominator work shown.

Correct response: $\frac{3}{5}$ is greater (common denominator 35: $\frac{21}{35}$ vs. $\frac{20}{35}$).

Likely reasoning: may indicate the fraction with the larger denominator is being chosen with no comparison method shown, the book's own diagnostic trigger for this error

Confirm (approach): Give a second comparison pair where the larger-denominator fraction happens to be smaller. If chosen again with no method shown, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Which is greater: $\frac{4}{9}$ or $\frac{5}{11}$?

Correct response: $\frac{5}{11}$ (common denominator 99: $\frac{44}{99}$ vs. $\frac{45}{99}$).

Teach: Always compare using a method: a common denominator, matching numerators, or a benchmark like one-half, never by eye.

Fresh Check (Recalculation Check)

Item: Which is greater: $\frac{5}{8}$ or $\frac{7}{12}$?

Correct response: $\frac{5}{8}$ (common denominator 24: $\frac{15}{24}$ vs. $\frac{14}{24}$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two recipes use $\frac{3}{8}$ cup and $\frac{2}{5}$ cup of sugar, and a baker wants to know which uses more, with no fraction-comparison method suggested.

Correct response: $\frac{2}{5}$ cup uses more (common denominator 40: $\frac{15}{40}$ vs. $\frac{16}{40}$).

Route: If check passes: move to NS-FR-02. If check fails: reteach requiring a shown comparison method on every item.

NS-FR-02 Build equivalent fractions by scaling both parts by the same factor

Mathematical claim: An equivalent fraction is built by multiplying (or dividing) both the numerator and denominator by the identical factor; changing only one part breaks the equivalence.

Prerequisites: NS-FR-01

Route Finder

Item: Write a fraction equivalent to $\frac{2}{5}$ with a denominator of 20.

Correct response: $\frac{8}{20}$ (multiply both parts by 4).

Tripwire

Observed: A student rewrites $\frac{3}{4}$ with a denominator of 12 and writes $\frac{3}{12}$.

Likely wrong response: $\frac{3}{12}$, from multiplying only the denominator by 4 and leaving the numerator unchanged.

Correct response: $\frac{9}{12}$ (multiply both the numerator and denominator by 3).

Likely reasoning: *may indicate only one part of the fraction is being scaled to reach the target, the book's own diagnostic trigger for this error*

Confirm (approach): Give a second equivalent-fraction item with a different target denominator. If only one part is scaled again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Write a fraction equivalent to $\frac{5}{6}$ with a denominator of 24.

Correct response: $\frac{20}{24}$ (multiply both parts by 4).

Teach: Whatever is done to the denominator must be done to the numerator, and vice versa. State the scale factor explicitly before applying it to both.

Fresh Check (Recalculation Check)

Item: Write a fraction equivalent to $\frac{3}{7}$ with a denominator of 28.

Correct response: $\frac{12}{28}$ (multiply both parts by 4).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe needs to be scaled so that a $\frac{3}{8}$ -cup measurement becomes an equivalent amount out of 32 total parts, with no equivalent-fraction language used.

Correct response: $\frac{12}{32}$ (multiply both parts by 4).

Route: If check passes: move to NS-FR-03. If check fails: reteach naming the scale factor before applying it to both parts.

NS-FR-03 Simplify to the GCF and accept improper fractions as valid

Mathematical claim: Simplifying a fraction means dividing by the greatest common factor, not just any common factor; a result should be checked once more for a remaining common factor. An improper fraction is not an error, it is a valid, different form of the same value as its matching mixed number.

Prerequisites: NS-FR-02

Route Finder

Item: Simplify $\frac{18}{24}$ completely.

Correct response: $\frac{3}{4}$ (GCF = 6).

Tripwire

Observed: A student simplifies $\frac{12}{18}$ by dividing by 2, getting $\frac{6}{9}$, and stops there.

Likely wrong response: $\frac{6}{9}$, from dividing by the first common factor found instead of the GCF, without checking the result again.

Correct response: $\frac{2}{3}$ (GCF of 12 and 18 is 6, not 2; $\frac{6}{9}$ still shares a common factor of 3).

Likely reasoning: *may indicate a fraction is being divided by the first common factor found rather than the GCF, and the result isn't checked again, the book's own diagnostic trigger for this error*

Confirm (approach): Give a second fraction with a GCF that isn't the first common factor a student would likely try. If it stops early again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Simplify $\frac{20}{30}$ completely, and state whether $\frac{9}{4}$ is a valid final answer as written.

Correct response: $\frac{2}{3}$ (GCF=10). Yes, $\frac{9}{4}$ is a valid, correctly-formed answer as an improper fraction.

Teach: After simplifying, always check the result once more: does any number bigger than 1 still divide both parts?

Fresh Check (Recalculation Check)

Item: Simplify $\frac{16}{24}$ completely, and state whether $\frac{11}{4}$ needs to be rewritten as a mixed number to be a correct answer.

Correct response: $\frac{2}{3}$ (GCF = 8). $\frac{11}{4}$ is already a valid, correct answer as an improper fraction; it does not need to be rewritten unless asked for.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A survey shows $\frac{14}{21}$ of respondents prefer one option, and a report needs this in simplest form, with no simplification instructions given.

Correct response: $\frac{2}{3}$.

Route: If check passes: chapter secure, move to NS-DC-01 (Chapter 4). If check fails: reteach the check-again-after-simplifying habit and confirm improper fractions are accepted as valid.

Chapter 4 -- Decimal Sense and Decimal Operations

NS-DC-01 Multiply decimals by counting total decimal places, not aligning points

Mathematical claim: Multiplying decimals means ignoring the decimal points during the multiplication itself, multiplying as whole numbers, then placing the decimal by counting the total decimal places across both factors.

Route Finder

Item: Multiply: 2.4×1.3

Correct response: 3.12 (2 total decimal places).

Tripwire

Observed: A student multiplies 3.5×1.6 and writes 56.

Likely wrong response: 56, from counting only one factor's decimal place instead of adding both factors' decimal places together.

Correct response: 5.6 (2 total decimal places from both factors: one from 3.5, one from 1.6).

Likely reasoning: *may indicate decimal places are being counted from only one factor rather than both added together, the book's own diagnostic trigger for this error*

Confirm (approach): Give a second multiplication item with a different number of decimal places in each factor. If only one factor's places are counted again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Multiply: 0.6×0.5

Correct response: 0.30 (2 total decimal places).

Teach: Label each factor's decimal-place count separately, then add the two labels before placing the decimal in the answer.

Fresh Check (Recalculation Check)

Item: Multiply: 4.25×0.3

Correct response: 1.275 (3 total decimal places).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe calls for 2.5 servings of a dish that uses 1.4 cups of rice per serving, and a cook needs the total rice, with no decimal-multiplication instructions given.

Correct response: 3.5 cups.

Route: If check passes: move to NS-DC-02. If check fails: reteach labeling and adding decimal places from both factors.

NS-DC-02 Subtract decimals with correct placeholder zeros

Mathematical claim: A whole number being subtracted from or subtracting a decimal needs an explicit decimal point and matching placeholder zeros before lining up and subtracting.

Prerequisites: NS-DC-01

Route Finder

Item: Subtract: $8 - 3.25$

Correct response: 4.75 (write 8 as 8.00 first).

Tripwire

Observed: A student subtracts $5 - 0.46$ without rewriting 5 with matching decimal places first.

Likely wrong response: An answer with too few decimal places or a borrowing error, from not giving the whole number a decimal point and zeros first.

Correct response: 4.54 (write 5 as 5.00, then subtract).

Likely reasoning: may indicate a whole number is not being given matching decimal places before subtracting, the book's own diagnostic trigger for this error

Confirm (approach): Give a second whole-number-minus-decimal item with a different number of decimal places needed. If the placeholder step is skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Subtract: $15 - 6.75$

Correct response: 8.25.

Teach: Give every whole number a decimal point and the same number of decimal places as the other number before subtracting anything.

Fresh Check (Recalculation Check)

Item: Subtract: $12 - 4.6$

Correct response: 7.4.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A gift card starts with \$20 and a purchase of \$13.40 is made, and a shopper wants to know what's left, with no decimal-subtraction instructions given.

Correct response: \$6.60.

Route: If check passes: move to NS-DC-03. If check fails: reteach writing placeholder zeros on whole numbers before every subtraction.

NS-DC-03 Divide decimals by shifting both numbers together; estimate first

Mathematical claim: Dividing by a decimal requires multiplying both the divisor and the dividend by the same power of ten, not just the divisor; dividing by a number less than 1 makes the result bigger, not smaller. Estimating first catches errors of a factor of 10.

Prerequisites: NS-DC-01

Route Finder

Item: Divide: $8.4 \div 0.7$, and estimate first.

Correct response: 12 (both numbers $\times 10$: $84 \div 7 = 12$). Estimate: about $8 \div 1 = 8$, so 12 is reasonable for dividing by less than 1.

Tripwire

Observed: A student divides $6.3 \div 0.9$ and writes 0.7.

Likely wrong response: 0.7, from multiplying only the divisor by a power of ten and leaving the dividend unshifted.

Correct response: 7 (both numbers shifted by $\times 10$: $63 \div 9 = 7$).

Likely reasoning: may indicate only the divisor is being shifted to clear its decimal point, while the dividend is left unshifted, the book's own diagnostic trigger for this error

Confirm (approach): Give a second decimal-division item, checking whether both numbers are shifted together. If only one is shifted again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Divide: $5.4 \div 0.6$, and predict bigger or smaller than 5.4 first.

Correct response: 9; bigger than 5.4, since dividing by less than 1 increases the result.

Teach: Write both numbers side by side and shift the decimal point the same number of places on each, visibly matched, before dividing.

Fresh Check (Recalculation Check)

Item: Divide: $9.6 \div 0.8$, and predict whether the answer is bigger or smaller than 9.6 before calculating.

Correct response: 12; bigger than 9.6, since dividing by a number less than 1 always increases the result.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A board 7.2 meters long needs to be cut into pieces that are each 0.9 meters, and a carpenter wants to know how many pieces result, with no decimal-division instructions given.

Correct response: 8 pieces.

Route: If check passes: chapter secure, move to NS-IN-01 (Chapter 5). If check fails: reteach shifting both numbers together and predicting bigger/smaller before dividing.

Chapter 5 -- Integer Sense

NS-IN-01 Order and compare integers using a number line, not digit size

Mathematical claim: For negative numbers, the one closer to zero (farther right on a number line) is greater, regardless of which digit looks bigger; zero itself is neither positive nor negative.

Route Finder

Item: Which is greater: -8 or -3?

Correct response: -3 (it sits closer to zero, farther right on the number line).

Tripwire

Observed: A student compares -13 and -6 and says -13 is greater because 13 is a bigger digit.

Likely wrong response: -13, from assuming a bigger digit means a bigger negative number.

Correct response: -6 is greater. It sits closer to zero, so it is farther right on the number line.

Likely reasoning: *may indicate a bigger digit is being read as a bigger negative number, the book's own diagnostic trigger for this error*

Confirm (approach): Give a second negative-integer comparison with different digits. If the bigger digit is chosen again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Order from least to greatest: -4, -11, -1

Correct response: -11, -4, -1.

Teach: Mark both numbers on a number line and read which one sits farther right, regardless of digit size.

Fresh Check (Recalculation Check)

Item: Order from least to greatest: -9, -2, -6. Is zero positive, negative, or neither?

Correct response: -9, -6, -2. Zero is neither positive nor negative.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two cities report temperatures of -15 degrees and -8 degrees, and a forecaster asks which is colder, with no integer-comparison language used.

Correct response: -15 degrees is colder (farther from zero on the negative side, i.e., the lesser value).

Route: If check passes: move to NS-IN-02. If check fails: reteach with a physical or drawn number line for every comparison.

NS-IN-02 Distinguish absolute value from opposite; absolute value is never negative

Mathematical claim: Opposite flips the sign and keeps the same distance from zero; absolute value drops the sign entirely and reports only the distance, which can never be negative.

Prerequisites: NS-IN-01

Route Finder

Item: Find the absolute value of -9, and the opposite of -9.

Correct response: $|-9| = 9$; the opposite of -9 is 9.

Tripwire

Observed: A student is asked for the absolute value of -7 and writes $|-7| = -7$.

Likely wrong response: -7, from keeping the negative sign in the absolute-value result.

Correct response: $|-7| = 7$ -- absolute value measures distance, and distance cannot be negative.

Likely reasoning: may indicate absolute value is being confused with the original signed value; any negative number given as an absolute-value answer is the book's own diagnostic trigger for this error

Confirm (approach): Give a second absolute-value item paired with an opposite item for the same number, checking both are answered correctly and distinctly. If confused again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Find the opposite of 12 and the absolute value of -12.

Correct response: Opposite of 12 is -12. $|-12| = 12$.

Teach: Absolute value strips the sign away entirely and gives only the distance from zero; it can never be negative, no matter what sign was inside the bars.

Fresh Check (Recalculation Check)

Item: Which has the greater absolute value: -10 or -6? Which has the greater value?

Correct response: -10 has the greater absolute value ($10 > 6$). -6 has the greater value (closer to zero).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A submarine is at a depth described as -85 meters, and a report asks how far below the surface it is, with no absolute-value language used.

Correct response: 85 meters below the surface (the absolute value of -85).

Route: If check passes: move to NS-IN-03 (Chapter 6). If check fails: reteach the sign-strips-away rule for absolute value specifically.

Chapter 6 -- Integer Operations

NS-IN-03 Add and subtract integers by rewriting subtraction as adding the opposite

Mathematical claim: Same-sign addition adds the sizes and keeps that sign (two negatives added get more negative, not positive); different-sign addition subtracts the smaller size from the larger and keeps the larger's sign; every subtraction is rewritten as addition of the opposite first, with the operation and the sign flipped separately.

Prerequisites: NS-IN-02

Route Finder

Item: Calculate: $-5 + (-3)$

Correct response: -8 (same signs: add the sizes, keep the negative sign).

Tripwire

Observed: A student calculates $7 - (-5)$ by dropping the negative sign and subtracting, getting 2.

Likely wrong response: 2, from dropping the negative sign instead of rewriting the subtraction as addition of the opposite.

Correct response: 12 (rewrite as $7 + 5$).

Likely reasoning: may indicate subtraction of a negative is being handled by dropping the sign rather than rewriting as addition of the opposite, the book's own diagnostic trigger for this error

Confirm (approach): Give a second subtraction-of-a-negative item with different numbers. If the sign is dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Calculate: $-7 + (-4)$, and $10 - (-3)$

Correct response: -11; and 13.

Teach: Say the rewrite out loud before calculating: 'subtracting a negative becomes adding a positive.' Change the operation and flip the second number's sign as two separate, checked steps.

Fresh Check (Recalculation Check)

Item: Calculate: $-9 + 4$, and $-6 - (-9)$

Correct response: -5 (different signs: subtract 4 from 9, keep the negative); 3 (rewritten as $-6 + 9$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A hiker starts at an elevation of -20 m relative to a trailhead and descends another 15 m, and a report asks the new elevation, with no addition-of-integers language used.

Correct response: -35 m.

Route: If check passes: move to NS-IN-04. If check fails: reteach the rewrite-as-addition-of-the-opposite step explicitly, saying it aloud each time.

NS-IN-04 Apply the correct sign rule per operation; recognize division by zero is undefined

Mathematical claim: 'Two negatives make a positive' applies only to multiplication and division, never to addition; brackets in a rewritten expression need each sign change checked separately; division by zero has no answer at all, since no number makes that division true.

Prerequisites: NS-IN-03

Route Finder

Item: Calculate: $-4 \times (-6)$

Correct response: 24 (two negatives multiplied give a positive).

Tripwire

Observed: A student calculates $-8 + (-3)$ and writes 24, applying the multiplication sign rule to addition.

Likely wrong response: 24, from applying 'two negatives make a positive' to an addition problem instead of multiplication or division.

Correct response: -11 (same signs added get bigger and stay negative, not positive).

Likely reasoning: may indicate the multiplication/division sign rule is being applied to addition, the book's own diagnostic trigger for this error

Confirm (approach): Give a second same-sign addition item immediately after a same-sign multiplication item, checking whether the correct rule is applied to each. If the rules are swapped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Calculate: $-6 \times (-3)$, and $-9 + (-9)$

Correct response: 18; and -18.

Teach: Name which operation is in front of you before picking a sign rule: addition and subtraction use one rule, multiplication and division use a completely different one.

Fresh Check (Recalculation Check)

Item: Calculate: $-12 \div 0$, and $-15 \div (-3)$

Correct response: $-12 \div 0$ is undefined (no answer exists). $-15 \div (-3) = 5$.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A scoring system subtracts points for wrong answers, and a student with -4 points per question over 5 questions asks for the total, with no multiplication-of-integers language used.

Correct response: -20 points.

Route: If check passes: chapter secure, move to NS-RN-01 (Chapter 7). If check fails: reteach naming the operation before applying any sign rule, and confirm division by zero is undefined, not zero.

Chapter 7 -- Rational Numbers

NS-RN-01 Classify numbers as rational correctly, including integers and repeating decimals

Mathematical claim: A rational number is any number that can be written as one integer over another; every integer qualifies (written over 1), and repeating or terminating decimals qualify, but a non-repeating, non-terminating decimal does not.

Route Finder

Item: Is -4 a rational number? Is 0.121212... (repeating) rational?

Correct response: Yes -- $-4 = -\frac{4}{1}$. Yes -- 0.121212... equals a fraction of two integers ($\frac{4}{33}$).

Tripwire

Observed: A student says 8 is not rational because 'it isn't a fraction.'

Likely wrong response: Not rational, from assuming rational numbers must already be written as fractions.

Correct response: 8 is rational -- every integer can be written over 1 ($\frac{8}{1}$), which makes it rational.

Likely reasoning: may indicate an integer is being excluded from the rational numbers because it isn't already written as a fraction, the book's own diagnostic trigger for this error

Confirm (approach): Give a second integer and ask whether it's rational, in a different numeric context. If excluded again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Is -6 rational? Is 3.14159265358979... (non-repeating) rational?

Correct response: Yes, $-6 = -\frac{6}{1}$. No, a non-repeating, non-terminating decimal is irrational.

Teach: Every integer can be written over 1. That alone makes it rational, regardless of whether a fraction is visibly shown.

Fresh Check (Recalculation Check)

Item: Is 0.7070070007... (never repeating in a fixed pattern) rational? Is -12 rational?

Correct response: No, a non-repeating, non-terminating decimal is irrational. Yes, $-12 = -\frac{12}{1}$ is rational.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A measurement is recorded as exactly 5 (a whole number of units), and someone asks whether it counts as one of the numbers that can be written as a fraction, with no rational-number vocabulary used.

Correct response: Yes -- $5 = \frac{5}{1}$, so it's rational.

Route: If check passes: move to NS-RN-02. If check fails: reteach the integer-over-1 rule and the repeating-vs-non-repeating distinction.

NS-RN-02 Compare rational numbers in different forms by converting to the same form first

Mathematical claim: Comparing a fraction and a decimal (especially negative ones) requires converting both to the same form first; digit size across two different forms means nothing on its own, and a conversion should be checked by converting back.

Prerequisites: NS-RN-01

Route Finder

Item: Compare: $\frac{3}{4}$ and 0.7

Correct response: $\frac{3}{4}$ is greater ($\frac{3}{4} = 0.75$, which is greater than 0.7).

Tripwire

Observed: A student compares $-\frac{2}{5}$ and -0.35 by looking at the digits $\frac{2}{5}$ and 0.35 directly.

Likely wrong response: An answer based on comparing the digits directly, with no conversion to a common form shown.

Correct response: -0.35 is greater ($-\frac{2}{5} = -0.4$, and -0.35 is closer to zero than -0.4).

Likely reasoning: may indicate a fraction and a decimal are being compared without converting to the same form first, the book's own diagnostic trigger for this error

Confirm (approach): Give a second negative fraction-vs-decimal comparison with different values. If compared without conversion again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Compare: $\frac{5}{6}$ and 0.8

Correct response: $\frac{5}{6}$ is greater ($\frac{5}{6} \approx 0.833$, which is greater than 0.8).

Teach: Convert both values to the same form first, always, before comparing anything. Check any conversion by converting back to confirm the value didn't shift.

Fresh Check (Recalculation Check)

Item: Compare: $-\frac{5}{8}$ and -0.6, and verify your conversion by converting back.

Correct response: -0.6 is greater ($-\frac{5}{8} = -0.625$, and -0.6 is closer to zero). Verify: -0.625 converts back to $-\frac{5}{8}$.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two discounts are advertised as $\frac{3}{8}$ off and 0.4 off, and a shopper wants to know which is the bigger discount, with no conversion method suggested.

Correct response: 0.4 off is the bigger discount ($\frac{3}{8} = 0.375$, which is less than 0.4).

Route: If check passes: chapter secure, move to NS-OP-01 (Chapter 8). If check fails: reteach requiring a conversion step and a round-trip check before any comparison stands.

Chapter 8 -- Order of Operations

NS-OP-01 Apply the correct order: brackets, exponents, then multiplication/division, then addition/subtraction, all left to right

Mathematical claim: Multiplication and division share equal priority and are done left to right, not multiplication always before division; addition and subtraction share equal priority and are done left to right, not addition always before subtraction.

Route Finder

Item: Evaluate: $24 \div 4 \times 2$

Correct response: 12 (division and multiplication share equal priority, work left to right: $24 \div 4 = 6$, $6 \times 2 = 12$).

Tripwire

Observed: A student evaluates $30 \div 5 \times 4$ by multiplying first, treating it as $30 \div (5 \times 4)$.

Likely wrong response: 1.5, from multiplying 5×4 first, treating multiplication as having higher priority than division.

Correct response: 24 (work left to right: $30 \div 5 = 6$, $6 \times 4 = 24$).

Likely reasoning: may indicate multiplication is being treated as automatically higher priority than division, the book's own diagnostic trigger for this error

Confirm (approach): Give a second mixed multiplication/division item in a different left-to-right order. If multiplication is done first again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Evaluate: $16 - 5 + 3$

Correct response: 14 (addition and subtraction, left to right: $16 - 5 = 11$, $11 + 3 = 14$).

Teach: Multiplication and division share equal priority: read left to right, the same rule for addition and subtraction.

Fresh Check (Recalculation Check)

Item: Evaluate: $8 + 5 - 6$, and explain why it is not calculated as $8 - (5 + 6)$.

Correct response: 7 (addition and subtraction share equal priority, left to right: $8 + 5 = 13$, $13 - 6 = 7$). Subtraction is not automatically done last or grouped separately.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe scaling calculation is written as 40 divided by 8 times 3, and a cook needs the result, with no order-of-operations language used.

Correct response: 15 ($40 \div 8 = 5$, $5 \times 3 = 15$).

Route: If check passes: move to NS-OP-02. If check fails: reteach that same-tier operations always go left to right, never by type priority.

NS-OP-02 Evaluate brackets and exponents before other operations

Mathematical claim: All work inside brackets is completed before anything outside them; an exponent is evaluated immediately after brackets, before any addition, subtraction, multiplication, or division.

Prerequisites: NS-OP-01

Route Finder

Item: Evaluate: $(9 - 4) \times 3$

Correct response: 15 (bracket first: $9 - 4 = 5$, then $5 \times 3 = 15$).

Tripwire

Observed: A student evaluates $5^2 - 9$ by subtracting first, getting $5^2 - 9$ treated as 5^2 computed after some other step.

Likely wrong response: An answer from adding, subtracting, multiplying, or dividing before evaluating the power.

Correct response: 16 (evaluate the exponent first: $5^2=25$, then $25-9=16$).

Likely reasoning: may indicate an exponent is being left until the end rather than evaluated right after brackets, the book's own diagnostic trigger for this error

Confirm (approach): Give a second item combining an exponent with another operation in a different arrangement. If the exponent is evaluated last again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Evaluate: $6^2 \div (2 + 1)$

Correct response: 12 (bracket first: $2+1=3$, then exponent: $6^2=36$, then $36 \div 3=12$).

Teach: After brackets, evaluate every exponent before any other operation. Treat the exponent step as non-negotiable, not optional or last.

Fresh Check (Recalculation Check)

Item: Evaluate: $(13 - 4) \times 2$, and $3^2 + 7$

Correct response: 18; and 16 ($3^2=9$ evaluated first, then $9+7=16$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A garden area calculation is written as a squared side length of 4, minus a walkway of 3 square meters, and someone needs the remaining area, with no order-of-operations language used.

Correct response: 13 square meters ($4^2=16$, then $16-3=13$).

Route: If check passes: chapter secure, move to NS-PW-01 (Chapter 9). If check fails: reteach the brackets-then-exponents priority explicitly.

Chapter 9 -- Powers and Exponents

NS-PW-01 Evaluate a power by repeated multiplication, not base times exponent

Mathematical claim: An exponent tells how many times the base appears as a factor in a multiplication, not a number to multiply the base by directly.

Route Finder

Item: Evaluate: 4^3

Correct response: 64 ($4 \times 4 \times 4$).

Tripwire

Observed: A student evaluates 5^3 as $5 \times 3 = 15$.

Likely wrong response: 15, from multiplying the base by the exponent instead of expanding the repeated multiplication.

Correct response: 125 ($5 \times 5 \times 5$).

Likely reasoning: may indicate the exponent is being multiplied directly by the base rather than expanded as repeated multiplication, the book's own diagnostic trigger for this error

Confirm (approach): Give a second power with a different base and exponent. If multiplied directly again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Evaluate: 3^4

Correct response: 81 ($3 \times 3 \times 3 \times 3$).

Teach: Expand the power fully as repeated multiplication before evaluating; the exponent counts factors, it is not a multiplier.

Fresh Check (Recalculation Check)

Item: Evaluate: 2^5

Correct response: 32 ($2 \times 2 \times 2 \times 2 \times 2$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A cell culture doubles in a pattern described as 2 multiplied by itself 6 times, and a researcher wants the total, with no exponent language used.

Correct response: 64.

Route: If check passes: move to NS-PW-02. If check fails: reteach expanding every power as repeated multiplication before evaluating.

NS-PW-02 Apply exponent rules correctly: zero exponent, negative bases, and same-base combining

Mathematical claim: Any non-zero base raised to the zero power equals 1, not 0; a negative sign is only part of the base if brackets enclose it, so -6^2 and $(-6)^2$ are different expressions; exponents combine by adding only when the bases match and are being multiplied, never when added or when bases differ.

Prerequisites: NS-PW-01

Route Finder

Item: Evaluate: 7^0 , and evaluate -3^2 and $(-3)^2$

Correct response: $7^0 = 1$. $-3^2 = -9$ (base is 3). $(-3)^2 = 9$ (base is -3).

Tripwire

Observed: A student evaluates both -4^2 and $(-4)^2$ and says both equal 16.

Likely wrong response: Both equal 16, from ignoring brackets and treating the negative sign as part of the base in both expressions.

Correct response: $-4^2 = -16$ (base is 4, then negated) and $(-4)^2 = 16$ (base is -4).

Likely reasoning: may indicate brackets around a negative base are being ignored, so the same answer is given whether or not brackets are present, the book's own diagnostic trigger for this error

Confirm (approach): Give a second bracket/no-bracket pair with a different base. If both are treated the same again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Evaluate: 5^0 , and -2^4 vs $(-2)^4$

Correct response: $5^0 = 1$. $-2^4 = -16$ (base is 2). $(-2)^4 = 16$ (base is -2).

Teach: Identify the base before calculating: if brackets enclose the negative sign, it's part of the base; if not, the exponent applies to the positive number first, then the negative sign is applied outside.

Fresh Check (Recalculation Check)

Item: A student writes $2^4 \times 5^3 = 10^7$. Explain why this is incorrect, then evaluate correctly.

Correct response: Bases differ (2 and 5), so exponents cannot combine. $2^4 = 16$, $5^3 = 125$, product = 2,000.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A student writes $3^2 \times 3^4 = 9^6$, and a peer asks if this is right, with no exponent-rule language used.

Correct response: No -- same base (3), exponents add: $3^2 \times 3^4 = 3^6 = 729$, not 9^6 .

Route: If check passes: move to NS-PW-03 (Chapter 10). If check fails: reteach identifying the base first, and combining exponents only for matching multiplied bases.

Chapter 10 -- Squares, Square Roots, and Estimation

NS-PW-03 Evaluate a square root as equal factors, not halving

Mathematical claim: A square root asks what number, multiplied by itself, gives the radicand; it is not found by dividing by 2, and factor pairs used must be equal, not just any two factors that multiply to the target.

Prerequisites: NS-PW-01

Route Finder

Item: Evaluate: $\sqrt{49}$

Correct response: 7 ($7 \times 7 = 49$).

Tripwire

Observed: A student evaluates $\sqrt{50}$ as 25, reasoning that $50 \div 2 = 25$.

Likely wrong response: 25, from treating square root as dividing the radicand by 2.

Correct response: $\sqrt{50}$ is between 7 and 8 (not a perfect square); $\sqrt{50} \neq 25$, since $25 \times 25 = 625$, not 50.

Likely reasoning: may indicate square root is being treated as halving rather than finding equal factors, the book's own diagnostic trigger for this error

Confirm (approach): Give a second non-perfect-square root, checking whether halving is attempted again. If it recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Evaluate $\sqrt{121}$, and explain why $\sqrt{40}$ is not 20.

Correct response: $\sqrt{121}=11$. $\sqrt{40}\neq 20$ since $20\times 20=400$, not 40; square root needs equal factors, not halving.

Teach: Ask for equal factors: what number, multiplied by itself, gives this value? Never divide the radicand by 2.

Fresh Check (Recalculation Check)

Item: Evaluate $\sqrt{81}$, and explain why $\sqrt{24}$ is not 6 even though $4\times 6=24$.

Correct response: $\sqrt{81} = 9$. $\sqrt{24} \neq 6$ because 4 and 6 are not equal factors; a square root needs a number multiplied by itself.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A square tile has an area of 36 square inches, and someone asks for the side length, with no square-root language used.

Correct response: 6 inches ($\sqrt{36}$).

Route: If check passes: move to NS-PW-04. If check fails: reteach requiring equal factors, never unequal factor pairs or halving.

NS-PW-04 Estimate a root using consecutive perfect-square benchmarks; recognize radicals give one answer

Mathematical claim: A non-perfect-square root is estimated between the nearest perfect square below and the nearest perfect square above the radicand, not any two perfect squares; a radical symbol gives the principal positive root only, never a plus-or-minus pair.

Prerequisites: NS-PW-03

Route Finder

Item: Estimate $\sqrt{70}$ between two consecutive whole numbers.

Correct response: $8 < \sqrt{70} < 9$ ($8^2=64$, $9^2=81$), approximately 8.4.

Tripwire

Observed: A student estimates $\sqrt{155}$ by writing $10 < \sqrt{155} < 14$.

Likely wrong response: $10 < \sqrt{155} < 14$, from not locating the two consecutive perfect squares closest to 155.

Correct response: $12 < \sqrt{155} < 13$ ($12^2=144$, $13^2=169$), approximately 12.4.

Likely reasoning: *may indicate the nearest consecutive perfect-square benchmarks are not being located, producing an interval wider than one whole number, the book's own diagnostic trigger for this error*

Confirm (approach): Give a second non-perfect-square root, checking whether the nearest consecutive perfect squares are used. If the interval is too wide again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Estimate $\sqrt{200}$ between two consecutive whole numbers, and state whether $\sqrt{49} = \pm 7$.

Correct response: $14 < \sqrt{200} < 15$ ($14^2=196$, $15^2=225$). $\sqrt{49} = 7$ only, not ± 7 .

Teach: Find the nearest perfect square below and the nearest perfect square above the radicand; the interval between their roots should be exactly one whole number wide.

Fresh Check (Recalculation Check)

Item: Estimate $\sqrt{90}$ between two consecutive whole numbers, and state whether $\sqrt{64}$ should be written as ± 8 .

Correct response: $9 < \sqrt{90} < 10$, approximately 9.5. $\sqrt{64} = 8$ only; a radical gives the principal positive root, never $\pm$.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A square garden plot has an area of 130 square meters, and a landscaper wants a whole-number estimate for the side length, with no square-root or benchmark language used.

Correct response: Between 11 and 12 meters ($11^2=121$, $12^2=144$), approximately 11.4.

Route: Chapter 10 and the full Book 1 skill map are complete once NS-PW-03 and NS-PW-04 are secure. Cross-book route: Book 2 (Algebra) Chapter 1 depends directly on the order-of-operations and signed-number fluency built here.